ABSTRACT

of dissertation for the Philosophy Doctor (PhD) degree in the educational program "8D05403 -Mechanics"

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THE PROBLEM OF N (N>2) PLANETARY BODIES WITH MASSES VARYING AT DIFFERENT RATES

General description of work. In this work, the N planetary problem of many spherically symmetric bodies with variable mass interacting according to Newton's law in a relative coordinate system is considered. Suppose that the masses of bodies vary isotropically at different rates in any law of mass variation, and that the mass of the parent star is much greater than the masses of the planets. During evolution, planets move in quasi-elliptical orbits that don't cross each other. Since the masses are variable, the differential equations of motion have assumed a non-autonomous form, and the task has become more complicated. Since there is no general solution to the Gylden-Meshchersky problem for any mass variation, the dynamics of the planetary system with varying masses was studied by methods of canonical perturbation theory. From the "*NASA* Exoplanet Archive", we know that about 87% of exoplanets with known eccentricities have values in the eccentricity range $0 \le e < 0.1$. The widely used methods for detecting exoplanets are the radiation velocity method and the transit method, and these methods are adapted to detect exoplanets orbiting on the plane about $i = 90^0$. If we consider the plane forming $i = 90^0$ as the main plane, there are many exoplanets that move in an orbit with almost zero inclination. Therefore, the canonical equation of motion was obtained using analogues of the second Poincarè system. These canonical equations best describe the dynamic evolution of celestial bodies moving in an orbit with low eccentricity and low inclination.

In order to express the perturbing function using canonical elements, the main part of the perturbing function was decomposed into a series up to the second degree inclusive by small parameters (an analogue of inclination and an analogue of eccentricity) using the *Wolfram Mathematica* symbolic system. A system of secular equations has been analytically obtained, which makes it possible to determine the change in the parameters of the orbits of planets in large time intervals. The secular system of equations consists of $4N$ linear non-autonomous equations and consists of two subsystems describing the shape of the orbit and the spatial position of the orbital plane. A dimensionless secular system of equations is obtained for any law of mass variation and for any N planet system. A system of dimensionless secular equations is explicitly written for the problem of four bodies and eight bodies with variable masses. In this work, we studied how the variability of the masses of celestial bodies affects the dynamic evolution of exoplanet system. In other words,

the evolution of the orbital elements in cases of variable mass was studied and the difference with the case of constant mass was analyzed.

Relevance of the dissertation theme. In classical mechanics, the problem of many bodies is investigated in case of constant mass, but the discovery and study of exoplanet systems shows that celestial bodies are non-stationary. The study of non-stationary problems in celestial mechanics has led to the need to take into account the variability of mass. For this reason, the study of the many body problem with variable mass is a topical task in celestial mechanics.

Since 1995, there have been more than 4 thousand exoplanet systems and about 6 thousand exoplanets have been discovered. In addition, more than 10 thousand celestial bodies are being considered-candidates for recognition of exoplanets. In the database of the “*NASA* Exoplanet Archive”, this data is updated and growing every day. In the next ten years, the number of exoplanets may reach tens of thousands. Exoplanet systems, which are growing day by day, also demonstrate the relevance of studying the N planetary problem of many bodies with variable masses in the astronomy. The study of the dynamic evolution of exoplanets makes it possible to determine the future stages of the development of the exoplanet system. Studying the dynamic evolution of non-stationary exoplanets will allow us to understand the process of the dynamic development of the Solar System and improve our understanding of the dynamic evolution in the galaxy.

The main goal of work. Obtaining secular equations of the perturbed problem of spherical many bodies in analogues of elements of the second Poincarѐ system, whose masses vary isotropically at different rates, and on the basis of the obtained equations, studying the dynamic evolution of non-stationary exoplanet systems.

Research tasks:

1. to consider the perturbed motion of the many body problem in the absolute coordinate system, the masses of which vary isotropically at different rates;
2. to study of canonical equations of motion in analogues of variables of the second Poincarѐ system in a relative coordinate system;
3. to build an algorithm for decomposing the perturbing function into a series with any accuracy using the *Wolfram Mathematica* and to decompose the perturbing function to the second degree;
4. to express of Laplace coefficients in terms of elliptic integrals of the first and second kind;
5. to obtain a system of secular equations as a result of averaging a system of canonical equations and to split into two subsystems;
6. to obtain systems of dimensionless secular equations that allow us to study the dynamic evolution of any N planetary system;
7. to express explicitly a system of dimensionless secular equations for four and eight body problems;
8. to solve the Cauchy problem by numerical methods for 1 stellar 3-planetary exoplanet system *K2-3*, 1 stellar 7-planetary exoplanet system *Trappist-1* and 2-planetary circumbinary system *TOI-1338*;
9. to study the evolution of the orbital elements of the exoplanets *K2-3*, *Trappist-1* and *TOI-1338* over a long period of time and to analyze the

results obtained to determine how mass variability affects the dynamics of the planetary system.

The object of the research. Non-stationary exoplanet systems with at least three planets, or binary systems with at least two planets moving in a quasi-elliptical orbit with low eccentricity and inclination.

Research methods. The methods of canonical perturbation theory and mathematical methods for solving non-autonomous linear differential equations and series expansion methods, modern numerical methods of *Wolfram Mathematica* and the Picard method were used.

Scientific novelty of the dissertation results. For the first time, secular equations of perturbations of the many body problem with isotropically varying masses have been obtained. The obtained secular equations are used to the study of the dynamic evolution of exoplanets with low eccentricity and inclination. These canonical equations are relevant for any law of mass varying and for an exoplanet system with any N planets.

The main provisions for the defense:

1. the system of canonical secular equations, obtained using analogues of the second Poincarѐ system, for the many body problem with isotropically varying masses in a relative coordinate system;
2. the system of dimensionless secular equations, obtained explicitly for the four-body problem and the eight-body problem;
3. dynamic evolution of changes in the orbital elements of the exoplanet systems *K2-3, Trappist-1* and *TOI-1338*.

The reliability and validity of the results. The results obtained in the dissertation work in the case of constant mass coincide with the well-known studies of other authors who studied the problem of many bodies in cases of constant mass. The reliability and validity of the obtained results are also confirmed by the presence of articles published in foreign peer-reviewed journals included in the *Web of Science (Clarivate Analytics)* and *Scopus* databases.

Theoretical and practical importance of the dissertation. The obtained secular equations using analogues of the second system of Poincarѐ variables can theoretically serve as a basis for studying the $N+1$ problem of a spherical bodies with variable masses. The obtained secular perturbation equations are of practical importance as a means of studying the dynamic evolution of exoplanets orbiting with low eccentricity and low inclination.

Approbation of the dissertation:

1. M. Minglibayev, A. Kosherbayeva. Evolution equations of the multi - planetary problem with variable masses // Abstract book of the "XXXIst General Assembly of the International Astronomical Union IAUGA 2022" Busan Exhibition and Convention Center (BEXCO) 55 APEC - ro, Haeundae - gu Busan, South Korea, August 02 - 11, 2022, p. 348.
2. M. Zh. Minglibayev, A. N. Prokopenya, A. B. Kosherbayeva. Investigation of the dynamic evolution of planetary systems with isotropically varying masses // Abstract book of the Complex Planetary Systems II Kavli-IAU Symposium 382, Namur, Belgium, July 3-7, 2023, p. 42-43.

3. Kosherbayeva A. Equations of secular perturbations of exoplanetary systems with variable masses // Abstract book of the "CELMEC VIII The Eighth International Meeting on Celestial Mechanics", University of Rome Tor Vergata, Italy, September 05-09, 2022, p. 10-11.
4. Mukhtar Minglibayev, Alexander Prokopenya, Aiken Kosherbayeva. The problem of many bodies with isotropically varying masses // Abstracts of the 28th International Conference on Applications of Computer Algebra ACA'2023. Warsaw University of Life Sciences (SGGW), Institute of Information Technology, Warsaw, Poland, July 17- 21, 2023, p. 70.
5. A. Chichurin, A. Prokopenya, M. Minglibayev and A. Kosherbayeva. Symbolic-Numeric Computation in Modeling the Dynamics of the Many-Body System TRAPPIST. // Abstracts of the 23rd International Conference on Computational Science, The Czech Technical University in Prague (CTU), Prague, Czech Republic, July 3-5, 2023, p. 99.
6. A.N.Prokopenya, M.Zh.Minglibayev, A.B.Kosherbayeva. Symbolic computation in studying the evolutionary equations in the problem of many bodies of variable masses. // Seminar on Computer Algebra, CMC faculty of MSU & CCAS Moscow, Russia, September 29, 2021, http://www.ccas.ru/sabramov/seminar/doku.php?id=archive2123.
7. M. Minglibayev, A. Kosherbayeva. Linear non-autonomous differential equations, determining secular perturbations of exoplanetary systems with variable masses // Abstract book of the International Conference «Computational and Information Technologies in Science, Engineering and Education» (CITech-2022) dedicated to the 90th anniversary of Academician N. K. Nadirov, to the 80th anniversary of Academician M. O. Otelbaev, Almaty, Kazakhstan, October 12-15, 2022, p. 92.
8. Kosherbayeva A. B.. Differential equations of planetary systems with variable masses // Materials of the International Conference "Annual international april mathematical conference in honor of the day of science workers of the Republic of Kazakhstan, dedicated to the 1150th anniversary of Abu Nasr al-Farabi and the 75th anniversary of the Institute of Mathematics and Mathematical Modeling", Almaty, Kazakhstan, April 01-03, 2020. p. 95-97.
9. Kosherbayeva A. B. Equations of motion of planetary systems with variable masses // Materials of International Scientific Conference of Students and Young Scientists «FARABI ALEMI» Almaty, Kazakhstan, April 6-9, 2020 p. 78.
10. Kosherbayeva A. B. Evolutionary equations of exoplanet systems with variable mass // Materials of International Scientific Conference of Students and Young Scientists «FARABI ALEMI» Almaty, Kazakhstan, April 6-8, 2022 p. 59.
11. A.N.Prokopenya, M.Zh.Minglibayev, A.B.Kosherbayeva. Symbolic-numeric computation in modeling the dynamics of the many-body systems of variable masses // Seminar on Computer Algebra, CMC faculty of MSU & CCAS, May 15, 2024. http://www.ccas.ru/sabramov/seminar/doku.php?id=start.

Publications. 18 scientific works have been published on the dissertation. Of these, 3 articles were published in the *Web of Science(Clarivate Analytics)* (Q1, Q3 and Q4) and *Scopus*(percentiles: 86, 18 and 2) database. 4 articles have been published in journals included in the list of publications recommended by the Committee for Quality Assurance in Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan, and 10 papers at international conferences. In general, contributions were presented at international conferences in South Korea, Italy, Russia, Poland, Belgium, the Czech Republic and Kazakhstan. One of them was presented at a conference organized by the International Astronomical Union at the XXXIst General Assembly in South Korea on the basis of winning a grant in 2022.

The personal contribution of the author. The main results of the dissertation were obtained by the author. The task statement and discussion of the results were carried out jointly with scientific consultants.

The scope and structure of the thesis. The dissertation consists of an introduction, 6 sections, a conclusion, a list of references, and an appendix.